# Lec 2, Report 1

sec 3

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**Q1: Prove That:**

$$K = \frac{\Pi\,|Poles|}{\Pi\,|zeros|}$$

## Solution

ch. eqn = $1 + G_1(S)\,H(S) = 0$

by grouping All K terms and deviding by non K terms we get the form

$$1 + K\,G_2(S) = 0$$

$G_2(S) = 0.L.T.F.$

$$\therefore G_2(S) = \frac{\Pi zeros}{\Pi Poles} = \frac{(S+z_1)(S+z_2)\cdots(S+z_n)}{(S+P_1)(S+P_2)\cdots(S+P_n)}$$

$$\therefore K = -\left[\frac{\Pi\ Poles}{\Pi\ zeros}\right]$$

$$\therefore |K| = \frac{\Pi\,|Poles|}{\Pi\,|zeros|}$$

$$\therefore K = \frac{\Pi\,|S+P_1||S+P_2|\cdots|S+P_n|}{\Pi\,|S+z_1||S+z_2|\cdots|S+z_n|}$$

$$\therefore K = \frac{\Pi\,|Poles|}{\Pi\,|zeros|}$$

**Example:**

$1 + G(S)\,H(S) = S^3 + KS^2 + 2S + K = 0$

① grouping all K terms

$$S^3 + 2S + K(S^2+1) = 0$$

Deviding By non K-terms

$$1 + K\,\frac{(S^2+1)}{S^3+2S} = 0$$

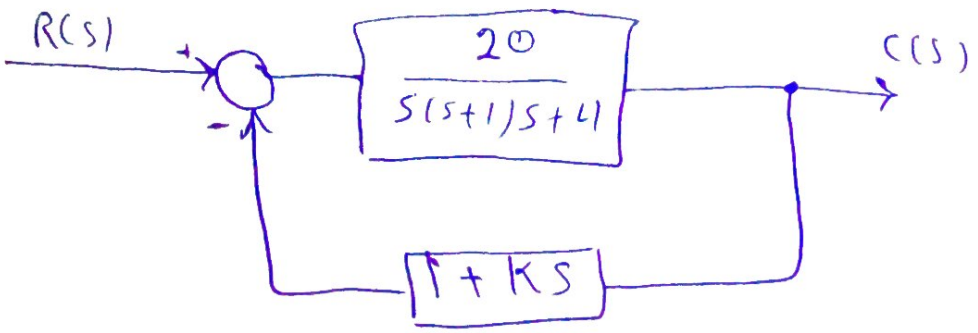
$$G_2(S) = \frac{S^2+1}{S^3+2S}$$

$$\therefore 1 + K\,G_2(S) = 0$$

$$\therefore K = \frac{\Pi\,|(S+j1.41)(S-j1.41)|}{\Pi\,|(S+j)(S-j)|}$$

$$K = \frac{\Pi\,|(Poles)|}{\Pi\,|(zeros)|}$$

**Q2: Sketch the root Locus and find the range of K for stability**



## Q2: Solution:-

$$O.L.T.F = G(S)H(S) = \frac{20(1+KS)}{S(S+1)(S+4)}$$

* ch. eqn $= 1 + G(S)H(S) = 1 + \frac{20(1+KS)}{S(S+1)(S+4)} = 0$

$$S(S+1)(S+4) + 20 + 20KS = 0$$

$$1 + \frac{20KS}{S(S+1)(S+4)+20} = 0 \qquad \text{Let } K' = 20K$$

$$\therefore O.L.T.F = K' \frac{S}{S(S+1)(S+4)+20}$$

* Poles: $S(S+1)(S+4)+20 = 0$

$\Rightarrow S_{1,2,3} = -5, \pm j2 \Rightarrow n_p = 3$

* Zeros: $S_z = 0 \Rightarrow n_z = 1$

* no. of asymptotes:

$n_p - n_z = 3 - 1 = 2$

* center of asymptotes:

$$\sigma_c = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{n_p - n_z} = \frac{-5}{2} = -2.5$$

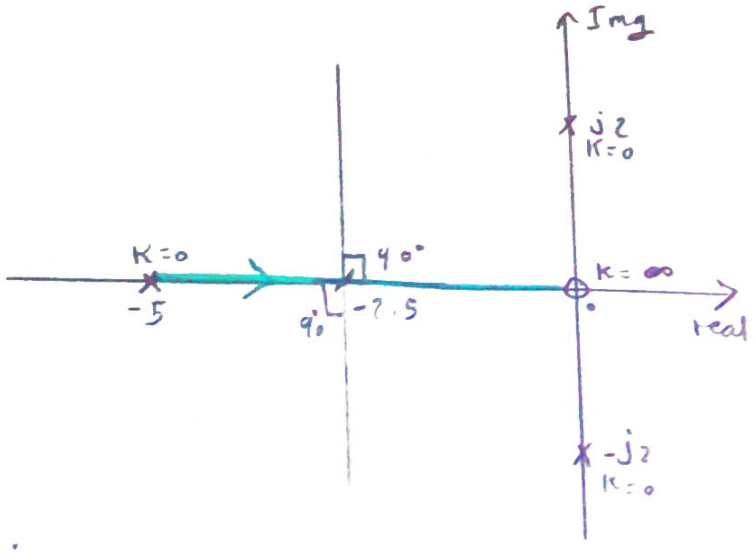
* $\theta = \frac{(2L+1)180}{2}$ $\Rightarrow$ at $L=0 \Rightarrow \theta_0 = 90°$

at $L=1 \Rightarrow \theta_1 = 270 = -90$

* range of K for stability

ch. eq: $S(S+1)(S+4)+20+K'S=0$

$\Rightarrow S^3 + 5S^2 + (4+K')S + 20 = 0$

| | | |
|---|---|---|
| $S^3$ | 1 | $4+K'$ |
| $S^2$ | 5 | 20 |
| $S^1$ | $\frac{20+K'-20}{5} > 0 \rightarrow ①$ | |
| $S^0$ | 20 | |

① $\frac{K'}{5} > 0 \Rightarrow \frac{20K}{5} > 0 \Rightarrow K > 0$